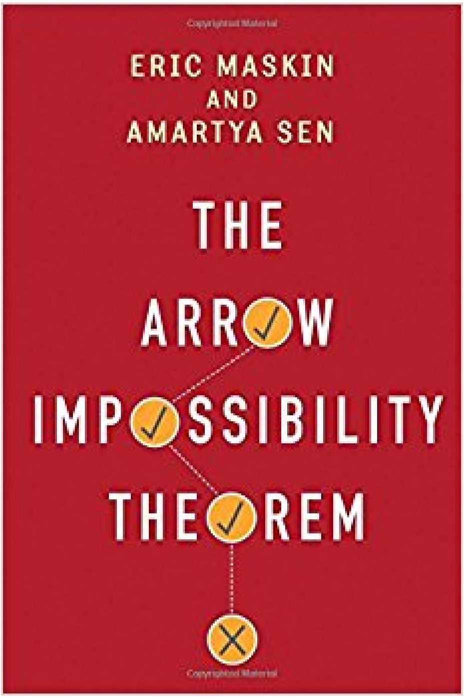
The Arrow Impossibility Theorem (Kenneth J. Arrow Lecture Series)



Synopsis

Kenneth J. Arrow’s pathbreaking "impossibility theorem" was a watershed innovation in the history of welfare economics, voting theory, and collective choice, demonstrating that there is no voting rule that satisfies the four desirable axioms of decisiveness, consensus, nondictatorship, and independence. In this book Eric Maskin and Amartya Sen explore the implications of Arrow’s theorem. Sen considers its ongoing utility, exploring the theorem’s value and limitations in relation to recent research on social reasoning, and Maskin discusses how to design a voting rule that gets us closer to the idealÃ¢â ¬â ¢given the impossibility of achieving the ideal. The volume also contains a contextual introduction by social choice scholar Prasanta K. Pattanaik and commentaries from Joseph E. Stiglitz and Kenneth J. Arrow himself, as well as essays by Maskin, Dasgupta, and Sen outlining the mathematical proof and framework behind their assertions.

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Customer Reviews

Without hyperbole, no postwar intellectual of the first rank has done more good for more peopleÃ¢â ¬â ¢above all, many of the world’s poorestÃ¢â ¬â ¢than Amartya Sen. (Boyd Tonkin The Independent)What is Arrow’s impossibility theorem? Why is it true? What are its implications for democratic decision making? Is its nihilism justified? These are the kinds of questions addressed in Maskin and Sen’s masterful Arrow lectures. These lectures and the accompanying essays provide an accessible introduction to Kenneth J. Arrow’s theorem for the neophyte and much food for thought for the cognoscente. (John A. Weymark, Vanderbilt University)How vital it is to understand the ideas behind Kenneth J. Arrow's impossibility theorem if we want to design reasonably fair ways of coming to consensus decisions that take equitable account of individual preferences. This book is a marvelous introduction to the theorem, a keystone in the theory of social choice. We are treated to a discussion of that theoryÃ¢â ¬â ¢its origin, background, and the challenges it points toÃ¢â ¬â ¢by some of its great architects. (Barry Mazur, Harvard University, author of Imagining Numbers)The pioneers of social choice theory give us lively, enjoyable, and stimulating lectures and exchanges of ideas. Their views, more than sixty years after the publication of Kenneth J. Arrow's theorem, are of paramount interest to anyone aware of the difficulties of collective decisions. (Marc Fleurbaey, Princeton University)

Eric Maskin is the Adams University Professor at Harvard University. He received the 2007 Nobel Memorial Prize in Economics (with L. Hurwicz and R. Myerson) for laying the foundations of mechanism design theory. He has also contributed to game theory, contract theory, social choice theory, political economy, and other areas of economics.Amartya Sen is the Thomas W. Lamont University Professor and Professor of Economics and Philosophy at Harvard University. In 1998 he was awarded the Nobel Memorial Prize in Economic Sciences, and in 1999 he was awarded the Bharat Ratna, India's highest civilian award. He is also a senior fellow at the Harvard Society of Fellows; distinguished fellow of All Souls College, Oxford; and a Fellow of Trinity College, Cambridge. His books have been translated into more than thirty languages.

Let's say all the boys in class decide to rank their favorite 20 cars.So every boy writes down on a piece of paper his own ranking of those 20 cars and then we go about listing how we would like to work it all out.0. Suppose we want our ranking to be transitive. So if I know that in the final ranking the Mercedes is above the Honda and the Honda is above the Fiat, then the Mercedes must be above the Fiat.1. Suppose that no matter how many boys there are in class, no matter how many of the (more than 20 factorial, to accommodate draws, missing cars etc.) permutations are submitted, you want to want to come up with a voting scheme that will deliver one, definitive, ranking. (a property of the scheme known us "Unrestricted Domain")2. Suppose that in this definitive ranking the Ferrari beats the Mercedes. And then suppose we take the Chevrolet out of the list. If then we repeat the vote without the Chevrolet but with all the other 19 cars, the Ferrari had better still beat the Mercedes, or else our voting scheme is unsatisfactory. (a property known as "Independence of Irrelevant Alternatives")3. Suppose that in every single boy's ranking the Daihatsu gets beaten by the Lamborghini. It had better also get beaten in the final ranking. (a property known as the "Pareto Principle")4. Suppose you don’t want there to be a boy who always prevails over the rest if he likes one car more than another (this is the "Non-Dictatorship Principle")The Arrow Impossibility Theorem says you’re out of luck. You can’t get all of the above.I guess everybody who’s been to elementary school already knows this, but Kenneth J Arrow gave mathematical proof. The proof’s rather easy to follow and I close this review with my version of it. The book is dedicated to an exposition of the Theorem and its ramifications.The result is not a big surprise, obviously, but it is the cornerstone of a beautiful theory. Armed with this result, other economists and philosophers have over the years looked at a number of "voting rules" such as the Anglo-Saxon "first past the post," the French runoff system, the plurality voting rule, ranking of candidates etc. and worked out when they will yield satisfactory results.This monograph of a book is written by some of the most prominent such theoreticians, including Amartya Sen, Eric Maskin and Partha Dasgupta, with short contributions from Joseph Stiglitz and Kenneth Arrow himself, all beautiful in their own way, though I must say I was confused by the introduction by Prasanta Pattanaik.Also, there is a full paper here that derives some very significant results concerning when "rank-order" voting "works well" (i.e. satisfies conditions such as the ones I describe above), when "plurality rule" voting "works well," when majority rule is decisive (answer: when there’s no "Concordet triplet" such that x>y>z for fewer than half the voters, y>z>x for fewer than half the voters and z>x>y for a third set of fewer than half the voters) and finally all this work yields the extremely powerful result that if, given a set of preferences, you can come up with some rule that "works well," then so will majority rule (and that therefore we have not wasted 200 years of democracy using this rule)That said, mathematical symbols are used when words would have fully sufficed. The complex math symbols are never, ever "pushed" in the proof. A Lebesgue integral is defined for no reason. (No use is ever made of measure theory anywhere past this definition) The author never says anything along the lines of "we recognize that this set is a group and apply Theorem X from group theory." It’s 100% math for the sake of math, and I found that annoying, especially since the book is riddled with errata.For example, on p. 112 there are extra brackets around the main expression that don’t belong; on p. 119 (and again on p. 120 and again on p.p. 143, 144) xRy and xRz and yRxRz and zRxRy are written with the second x’s and y’s and z’s as subscripts when the notation for "x dominates y under R" has been defined as xRy; on p. 123 we are assured that for some t

This book consists of expository lectures by Sen and Maskin on ArrowÃƒÂ¢Ã â ¬Ã â„¢s work, commentary by Arrow himself, and supplementary materials especially an introduction by Pattanaik and mathematical papers by Sen and Maskin. However the result would have been far more successful if the organizers had invited a talk by someone like Donald Saari who could have given a far deeper view of the famous Arrow theorem.At one point Sen even talks about the need to broaden ÃƒÂ¢Ã â ¬Ã Å“social choiceÃƒÂ¢Ã â ¬Ã Â• theory to include richer sources of information (pp. 76 ÃƒÂ¢Ã â ¬Ã â œ 80) ÃƒÂ¢Ã â ¬Ã â œ more than just the preferential or rank orderings assumed by the Arrow theorem and used in common voting methods like the Borda Count and Instant Runoff Voting. While a laudable goal, my feeling is that this needs a much different approach, one involving nonlinear agents or dynamics. Yet incredibly Sen fails to note that ArrowÃƒÂ¢Ã â ¬Ã â„¢s impossibility result is, in fact, a direct consequence of an assumption that drastically restricts the use of information that is already presents in the votersÃƒÂ¢Ã â ¬Ã â„¢ rankings of candidates. This is, of course, the axiom of ÃƒÂ¢Ã â ¬Ã Å“Independence of Irrelevant AlternativesÃƒÂ¢Ã â ¬Ã Â•, which I think might be better called the assumption of ÃƒÂ¢Ã â ¬Ã Å“Forbidden IntensityÃƒÂ¢Ã â ¬Ã Â• (FI), as it forbids the voting algorithm from using the intensity of a voterÃƒÂ¢Ã â ¬Ã â„¢s ranking of one candidate over another.To illustrate this concept, suppose I am a voter in an election with 4 candidates and I feel strongly that candidate x is by far the best and that candidate y is by far the worst, while I am not enthusiastic about either candidate a or b but would pick a over b if forced to. Thus I would rank these 4 candidates x > a > b > y. However FI says that the electoral ranking, or ÃƒÂ¢Ã â ¬Ã Å“social orderingÃƒÂ¢Ã â ¬Ã Â•, of these 4 candidates, as produced by the voting algorithm under consideration, should ignore this difference. That is, the electoral ranking of x and y should not be affected by how I rank a or b ÃƒÂ¢Ã â ¬Ã â œ that such rankings are ÃƒÂ¢Ã â ¬Ã Å“irrelevantÃƒÂ¢Ã â ¬Ã Â• even though the way I rank a and b in between x and y is the way I express the strong intensity of my opinion, or the weakness of this intensity when I rank no candidates between a and b.In other words, FI forces the voting algorithm to ignore very important information provided by the voters. The intensity of a ranking of x over y is easily quantified, using a variation on SaariÃƒÂ¢Ã â ¬Ã â„¢s concept (pp. 189 - 191," Decisions and Elections", 2001), by defining Iv(x,y) = k if x > a1>ÃƒÂ¢Ã â ¬Ã Â¦ ak-1>y when voter v ranks k-1 candidates (a1,ÃƒÂ¢Ã â ¬Ã Â¦,ak-1) between x and y. In addition set Iv(x,x) = 0 and Iv(x,y)= -Iv(y,x) if x < y. Then Saari shows that the Borda Count satisfies all the Arrow axioms if FI is modified to permit the voting algorithm to use this intensity function (which implies voter transitivity). This modification Saari calls ÃƒÂ¢Ã â ¬Ã Å“Intensity of Binary IndependenceÃƒÂ¢Ã â ¬Ã Â• but IÃƒÂ¢Ã â ¬Ã â„¢d prefer to call it the assumption of ÃƒÂ¢Ã â ¬Ã Å“Permitted IntensityÃƒÂ¢Ã â ¬Ã Â• (PI).In fact, ArrowÃƒÂ¢Ã â ¬Ã â„¢s theorem more properly applies to pairwise comparisons (showing the severe limitations of that approach to voting, in line with the Condorcet paradox) rather than rank ordering, where SaariÃƒÂ¢Ã â ¬Ã â„¢s theorem hints at the fundamental role of the Borda Count. ArrowÃƒÂ¢Ã â ¬Ã â„¢s theorem is also a demonstration of the limitations of the axiomatic approach to voting algorithms, in that some properties which may seem natural from one point of view may have hidden consequences which require a much broader perspective. That perspective should focus more on the practical aspects of voting than on theoretically undesirable properties which may rarely come into play or which can be anticipated and guarded against by appropriate procedures.For example, as an activist mathematician I often recommend that the Borda Count be used in the form ÃƒÂ¢Ã â ¬Ã Å“rank your top 3 choicesÃƒÂ¢Ã â ¬Ã Â• (or sometimes 4 or 5), to reduce the cognitive workload of the voter, the vote counting work load, and tactical voting. This seems to be by far the best method in most informal or low key voting situations, such as when a group wants to rank action plans in a way that builds consensus. Combining it with Peter EmersonÃƒÂ¢Ã â ¬Ã â„¢s ÃƒÂ¢Ã â ¬Ã Å“modified Borda CountÃƒÂ¢Ã â ¬Ã Â• is a good way to discourage single choice voting. In more partisan or large scale situations, a danger is that a faction may run ÃƒÂ¢Ã â ¬Ã Å“cloneÃƒÂ¢Ã â ¬Ã Â• candidates or superfluous alternatives to subvert the ranking of true alternatives. In this case, a primary election may eliminate the clones, or in some cases proportional representation may be used to identify different factions and their top candidates (IÃƒÂ¢Ã â ¬Ã â„¢ve developed my own clustering algorithm for this purpose).Instead of this kind of practical work, Maskin tries to rescue FI by identifying restrictions to the rankings which will satisfy FI. He cites BlackÃƒÂ¢Ã â ¬Ã â„¢s ÃƒÂ¢Ã â ¬Ã Å“single peakednessÃƒÂ¢Ã â ¬Ã Â• criterion, which guarantees a Condorcet winner. Then he defines simple, but impossible to enforce, criteria for Borda and Plurality voting to satisfy FI. However I see FI as the problem, not the solution, because of the way it refuses to admit important information. And how would you restrict how people rank candidates in any kind of legal, let alone moral, way?There are also some issues with the mathematics. First of all, SenÃƒÂ¢Ã â ¬Ã â„¢s definition of decisiveness on p. 35 is not clear. His wording says that for a particular voting algorithm a set of voters G is decisive for a pair of candidates if whenever all voters in G rank x > y, then x > y in the electoral ranking. The problem here is that uses set notation (is not ordered) but the notation x > y specifies an ordering. So should the set be replaced by the ordered pair (x,y)?The answer is ÃƒÂ¢Ã â ¬Ã Å“NoÃƒÂ¢Ã â ¬Ã Â• because then the proof of the Spread of Decisiveness Lemma fails. That is, the way to prove an ordered pair version of the Lemma, following SenÃƒÂ¢Ã â ¬Ã â„¢s sketch, would be to assume that (a,b) is an arbitrary ordered pair such that all voters in G rank a > b. Then show that the electoral ranking must specify a > b, given that G is decisive for some ordered pair (x,y).However consider the case (a,b) = (y,x). Then the decisiveness of (x,y) tells us nothing about (a,b) since the hypothesis fails; that is, x > y for all voters in G whereas we need y > x. If constitutes 3 voters with a = y, we encounter a similar problem. WeÃƒÂ¢Ã â ¬Ã â„¢d want to show a > b given x > y by decisiveness, but the method of proof gives us instead b > x > y=a.Hence the correct definition of decisiveness is that G is decisive for as a set if it decisive for both orderings. That is, if x > y for all voters in G, then x > y electorally, or if y > x or all voters in G, then y > x electorally. This concept is easily generalized to any set X of size 2 or more by saying that G is decisive for X as a set if G is decisive for each ordering of X.Then SenÃƒÂ¢Ã â ¬Ã â„¢s proof works if does not intersect . For all voters ranking a > b, which includes all voters in G, we can use FI to transform any ranking of into the canonical ranking a > x > y > b first by switching the ranking of x, then the ranking of y, without affecting the ranking a > b. For all remaining voters we can switch x until a > x and y until y > b, getting y > b > a > x. In both cases a > x and y > b for all voters, so a > x > y > b electorally using Pareto for a > x and y > b and decisiveness for x > y. Hence a > b electorally by transitivity. Likewise we get b > a electorally if b > a for all voters in G using the decisiveness of (y,x) for G, transforming all b > a orderings to the canonical ordering b > y > x > a and the remaining orderings to x > a > b > y.However, in the case of one of a or b = x or y, separate proof is needed. Take a = x for example and first consider the case that a > b, including all voters in G. Then we can switch a voterÃƒÂ¢Ã â ¬Ã â„¢s ranking of a=x > b > y or of y > a=x > b into the canonical ranking a=x > y > b by FI. For the remaining voters we could switch a ranking of b > a=x > y or of b > y > a=x into the canonical ranking y > b > a=x. Thus we get y > b for all voters, so that y > electorally by Pareto, which when combined with x > y electorally by decisiveness yields a > b electorally by the electoral transitivity of a=x > y > b. If instead we want to prove b > a electorally, weÃƒÂ¢Ã â ¬Ã â„¢d end of up using the decisiveness of y > x by reduction to the canonical ordering b > y > a=x.This lemma demonstrates the incredible power of the assumption of Forbidden Intensity, in fact how it is way too powerful for its own good. ItÃƒÂ¢Ã â ¬Ã â„¢s too bad that these economists have not been talking to mathematicians like Saari, who have moved far ahead in their analysis.

This is a book that should be read and used in conjunction with the works of Elliott Jacques.

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